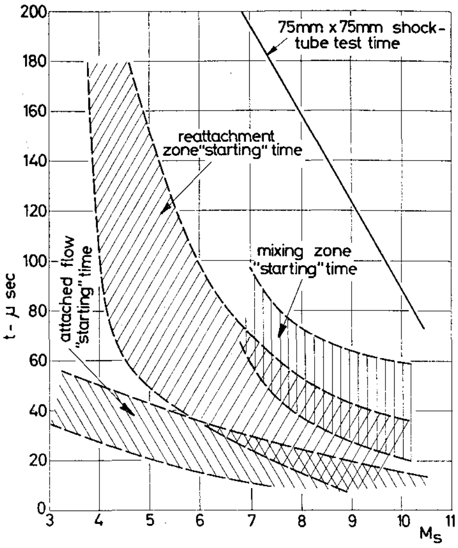

Fig. 1 Time of establishment of steady flow in various flow regions over a backward facing step in the shock tube.

ever, because of the fact that the establishment of steady heat transfer conditions in a separated region is longer than the corresponding attached flow case, there seems to be a feeling that this time may be much longer than available shock tube testing duration. A study, related to this problem, on the adjustment of a separated flow to transient external flows has been conducted by Ihrig and Korst.[6] According to Ref. 6 the adjustment of a separated flow to transient conditions is characterized by three "characteristic times." 1) pressure wave characteristic time $\Delta t_a = L/a_a$; 2) mass transfer characteristic time $\Delta t_m = (L/U_a\Gamma)$; and 3) heat transfer characteristic time $\Delta t_H = (L/U_a\, St)$, where L is a characteristic length of the separated region, a_a = local speed of sound, U_a = local external velocity, Γ = mass mixing parameter, and St = Stanton number. It is realized that of these three characteristic times the slowest adjustment process is the establishment of steady heat transfer. Assuming turbulent heat transfer values of $St \sim 0.01$, one finds that $\Delta t_H = (100/M_a)\cdot \Delta t_a = [100 \cdot L/M_a\, a_a]$ where $M_a = U_a/a_a$ = local external Mach number. Let the product $M_a \cdot a_a$ be approximately equal to the free shock tube flow value. This can be justified by the fact that the decrease of a_a behind the expansion fan at the separation point of a base type flow is followed by a comparable increase of M_a. The values of Δt_H for a 2-mm separation region height is then about 190 μsec at $M_s = 4$ and about 70 μsec at $M_s = 10$. These values are not out of the range of testing duration of moderate size shock tubes. However, according to this estimation, studies of laminar heat transfer in separated flow may require much longer testing duration due to the lower values of laminar heat transfer rates. This analysis of Ihrig and Korst[6] is applicable to the case of abrupt changes in external flow conditions such as jump in stagnation pressure during wind-tunnel tests. However, the conditions of flow establishment in the shock tube are different. Up to the arrival of the shock wave there is no flow and the model is at room temperature. The flow over the model is started impulsively immediately after the shock wave passage. With this impulsive start the model surfaces, including the separated region boundaries, are exposed to the high-temperature gas without the benefit of a protective viscous layer. So initially those surfaces are exposed to extremely high heat transfer rates. With the development of a viscous layer, be it a boundary layer or a separation region, the heat transfer rate is adjusted to an equilibrium value that is determined by the aerodynamic parameters. The initial adjustment duration will be much faster than those estimated by the quasi-steady characteristic times of Ref. 6 because of the much higher initial transient heat transfer rates due to the nonexistence of initial viscous layers. The initial process is somewhat similar to the heat transfer to the shock tube wall immediately following the shock front.

The establishment of equilibrium heat transfer rates in a laminar separated region over a backward facing step of 1.5-mm height in the shock tube is discussed in Ref. 5. The measured starting times of steady heat transfer rates at the various flow regions are shown in Fig. 1. It is found that for M_s above 5, steady heat transfer rates are established in the laminar separated regions two to three times slower than the corresponding heat transfer rates for attached laminar flows in the shock tube. Still, this steady heat transfer establishment time is about 40–60% of available testing duration in the 7.5 × 7.5 cm − 7-m-long shock tube. Laminar heat transfer rates can then be studied in a separated region with reasonable range of shock tube operating conditions. The establishment of steady heat transfer rates in turbulent separation regions is of course much faster than the corresponding laminar case. So that one may conclude that although careful attention must be paid to the establishment of steady heat transfer conditions in separated flows, there are ranges of shock tube operating conditions where such equilibrium is reached within the available test duration. The much longer duration of testing time in the tailored interface shock tunnel makes this facility suitable for most separated region studies including laminar heat transfer measurements.

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Comments

Comments on "A Note on the Classical Buckling Load of Circular Cylindrical Shells under Axial Compression"

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THE author's[1] photoelastic pictures of the buckling process are interesting, especially the concrete evidence that the shell passes through intermediate unstable equilibrium states

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before arriving at the usually observed diamond buckling pattern. His plans for further and better studies of this type may clear up some of the unanswered questions about the buckling process for cylinders in axial compression. In fact, the experimental determination of the true "buckling mode" may well require the use of this technique, since it is possible that the shell configuration undergoes significant changes after the inception of buckling and before deformations become visible to the eye or camera.

However, the author claims to have attained experimental buckling loads within 10% of the classical load and, on this basis, concludes that the classical load is attainable if the cylinder is free of initial imperfections. These statements deserve careful scrutiny. In the first place, it is evident from the different slopes of the theoretical and experimental curves of the author's Fig. 1 that the true extensional stiffness of the experimental cylinder material is somewhat greater than that used in the calculation of the classical load. If the experimental result is compared with the correct classical load for the cylinder specimen, the discrepancy between theory and experiment indicated in the author's figure is more than doubled to 16%; presumably, this figure is typical of the author's results. Secondly, the author's cylinder specimen has a relatively low ratio of radius to thickness ($R/t = 154$), and it is well known that buckling loads as high as the author's are attainable for such cylinders without unusual efforts to eliminate shape imperfections. For example, Fig. 1 is a plot of buckling load against R/t with the author's result shown by the solid circle. The hatched area encompasses a large body of data taken from the work of 14 different investigators and compiled in Ref. 2. These experiments were conducted under widely varying conditions on specimens, in the range $0.5 < L/R < 5$, made from a variety of materials. The open circles are additional recent results[3] obtained on 7075-T6 aluminum alloy cylinders with $L/R = 4$. Normal production methods were used in manufacturing these cylinders. The wide variation in the results shown in Fig. 1 may well be due partly to initial imperfections; however, the spread probably also is caused, in part, by variations in testing conditions. The salient fact is that the author's result does not constitute new or unique evidence of the importance of imperfections and certainly does not warrant his strong conclusion.

More convincing evidence of the importance of careful manufacture and testing is provided by the results[4] shown by the square symbols in Fig. 1. These were obtained from tests of copper cylinders with $L/R = 2.5$ which were made by carefully electroplating the shells on accurately machined wax mandrels. Radius variations in these shells were held to within half the thickness and thickness variations to 3%. The tests were conducted carefully with special attention to alignment. In contrast to the author's result, these results are seen to represent a clear and significant improvement over the general body of existing data. Moreover, additional tests (see Ref. 4) of cylinders with specified imperfections under carefully maintained uniform conditions have confirmed that imperfections alone can indeed cause decreases in strength of up to 40%.

However, even the results of Ref. 4 are not sufficient evidence to support the author's conclusion, for the author is completely disregarding an important source of error in the classical theory which is entirely unrelated to initial shape imperfections: the inconsistent assumption made in classical theory regarding end conditions. Whereas in classical theory it is tacitly assumed that the ends are free prior to buckling so that the cylinder retains its shape and uniform (membrane) stress up to the buckling load, the ends of a real cylinder actually are restrained (as in the author's experiments, for example) so that the prebuckling shape and stress state is the axisymmetric state calculated by Föppl.[5] Stein[6] has shown that axial compression buckling (in the classical sense) away from the Föppl prebuckling state may occur at

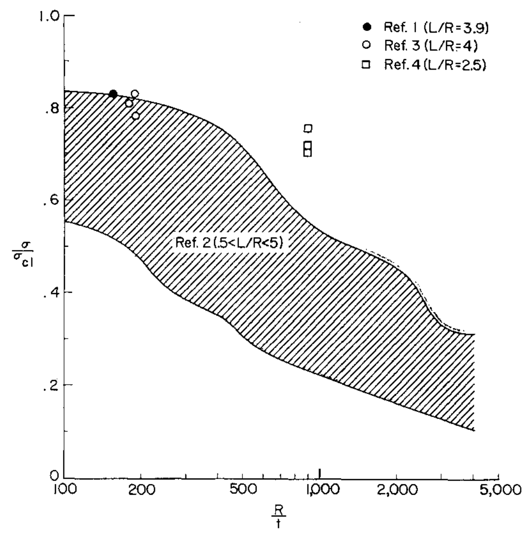

Fig. 1 Influence of radius-thickness ratio on buckling data for nominally perfect cylinders in axial compression.

about half the classical buckling load for one case of simply supported, initially perfect cylinders. Presumably, the reduction would be less for cylinders with clamped ends, but similar solutions for clamped cylinders are not yet available. The author's test specimen probably approximates a clamped case, but it appears highly unlikely that he will be able to achieve the classical buckling load merely by eliminating initial shape imperfections from his experimental cylinders.

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THE experimental results in Fig. 1 of the author's note[1] were plotted using the strain readings determined from photoelastic data taken at one location on the shell. The actual value of Young's modulus used in calculating the critical buckling load of the shell was that of the shell deter-

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